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Marketing Research Report No. 729

Managerial Aspects of Least-Cost Feed Formulation with Linear Programming

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PREFACE

This report is a study of quantitative management techniques for use in manufacturing formula feeds. It contains an appraisal of linear programming in feed formulation and the impact of frequent formula changes on ingredient usage rates. This study is a cooperative project of the Purdue Agricultural Experiment Station and the Economic Research Service of the U.S. Department of Agriculture.

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Washington, D.C. August 1965

SUMMARY AND CONCLUSIONS

When properly used, linear programming (LP) can be a valuable management tool for computing feed formulas, anticipating changes in ingredient usage rates, and developing feed specifications. Gross savings of $1.70 per ton were attributed to the use of LP during a 23-week test in two firms. Annual gross savings resulting from the use of LP were projected at $116,000 for firm A and $17,000 for firm B. These projections were based on average per ton savings and the average annual production of the six feeds studied.

Of the $1.70 per ton savings, $1.33 was attributable to the use of linear programming. The remaining $0.37 was the result of formulating the feeds every week rather than when management decided the formula might be too expensive. However, weekly formulation yielded maximum net savings for only one of the six feeds. At a formulation cost of $10, weekly formulations yielded maximum net savings for feeds produced only in volumes greater than 377 tons. Biweekly formulation is optimum for feeds produced in volumes between 85 and 377 tons per week.

Linear programming appears to be profitable for the small- and medium-sized manufacturer as well as the large manufacturer. Formulation every 4 or 5 weeks is the most profitable for small firms. At a formulation cost of $10, a review every 4 weeks was indicated for feeds produced in volumes between 26 and 39 tons per week, and a review every 5 weeks was indicated for feeds produced in volumes between 11 and 26 tons per week. However, at a formulation cost of $15, a review every 4 weeks was indicated for feeds manufactured in volumes between 39 and 58 tons per week. A review every 5 weeks was indicated for feeds produced in volumes between 16 and 39 tons per week.

Severe shortages or overstocking of ingredients would have occurred in several instances if successive LP formulas had been used. If potential savings resulting from the use of LP are to be obtained, methods must be devised to overcome problems posed by drastic changes in ingredient usage rates. Some alternative measures to minimize the severity of these fluctuations or their impacts are discussed. Though none of the proposals can completely alleviate the problem, the use of supplementary information from LP, which consists of price ranges and opportunity costs, can help management anticipate changes in usage rates. For ingredients that are close substitutes and whose prices move together (e.g. 44-percent and 50-percent soybean meals), the difference in the spread between their prices is a better indication of sensitivity than the upper or lower limits of price ranges.

Linear programming also makes it possible to measure the cost of each LP restriction or formula specfication. This information is a valuable guide for the nutritionist to use in altering specifications and pinpointing those areas most in need of nutrition research. The nutritionist, for example, should seek to narrow tolerances on the more costly specifications while retaining wide ones on the less costly specifications. Since the test data indicated that some of the specification costs were highly sensitive to weekly changes, a series of specification costs should be used to evaluate quality specifications.

Formulas may be programmed that are least-cost with respect to cost per unit of TDN or metabolizable energy instead of per unit of weight. For a complete dairy feed, the cost of TDN was reduced by 3.4 percent by minimizing the cost per unit of TDN instead of per unit of weight. However, through the use of LP information on the cost of the TDN or energy restriction for a formula minimizing the cost per unit of weight, it is possible, for all practical purposes, to obtain a formula that is least-cost per unit of TDN or metabolizable energy, as well as per unit of weight.

MANAGERIAL ASPECTS OF LEAST-COST FEED FORMULATION WITH LINEAR PROGRAMMING

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INTRODUCTION

This report gives the results of using linear programming for feed formulation in two firms during a 23-week period. Since its primary purpose is to illustrate the role of LP in improving managerial decisions and controls, it is written for management personnel rather than the technician preparing the LP problem. The following are specific topics discussed in this report:

1. Comparison of the cost of ingredients for LP formulas with that of ingredients for conventional formulas.
2. Selection of the optimum frequency of reviews for LP formulas.
3. Changes in ingredient usage rates resulting from the use of LP.
4. Use of cost ranges and opportunity costs for procurement planning.
5. Use of the cost of LP restrictions (shadow prices) for guiding the nutritionist in changing specifications.

Least-cost feed formulation is the process of finding the minimum cost of a combination of ingredients for a given set of specifications. These specifications contain both nutrient and nonnutrient requirements. The nutrient requirements set restrictions on the maximum and minimum amounts of nutrients--protein, TDN (total digestible nutrients), amino acids, calcium, or fat--to be used in various feeds. Nonnutrient requirements place restrictions on the maximum and minimum amounts of individual or groups of ingredients to be used for the purpose of obtaining the desired appearance, texture, and odor. Since LP makes it possible to compute the minimum cost from a set of specifications, LP formulas are least-cost.

In accordance with the prevailing practice in the feed manufacturing industry, all formulas in the body of this report are least-cost with respect to weight. Formulas, however, can be made least-cost with respect to other criteria, such as TDN or metabolizable energy. The impact on costs and the amount of ingredients used (usage rates) as a result of changing the least-cost criterion is discussed in the appendix.

SAVINGS RESULTING FROM THE USE OF LINEAR PROGRAMMING

Six feeds, three each from two cooperating firms, were formulated with LP for each week from January 14 to June 17, 1963. Firm A was a high-volume, single-plant firm in the Midwest, and firm B was a large multiplant operation from which an east coast mill was selected. Both firms had nutrition departments and formulated when they believed market conditions warranted their doing so. Though all LP formulas contained the specifications used by the firms, the formulation procedures of the firms were not influenced.

The cost of ingredients for LP formulas reviewed each week was $1.70 per ton less than the cost of those for formulas used by the firms during the study period (table 1). Of this $1.70 savings per ton, $0.37 resulted from weekly review of the formulas. The use of LP instead of conventional computing procedures accounted for the remaining savings of $1.33 per ton. Savings resulting from weekly review of feeds ranged from $0.16 to $0.84 per ton, in contrast to savings resulting from the use of LP, which ranged from $0.49 to $2.49 per ton. Annual gross savings resulting from weekly review with LP would have amounted to $116,000 for firm A and $17,000 for firm B.

Computing and changeover costs must be deducted to obtain net savings. The computing cost is the difference between the out-of-pocket computer expenses and hand-calculating expenses. Changeover costs are related to management procedures, tag restrictions, and mill technology. Both of these costs vary so much among firms and plants within firms that general cost estimates are not feasible. These costs, however, are small in contrast to the gross savings resulting from the use of LP.

Selection of the Optimum Period of Time between Reviews

Principles for determining the optimum period of time between reviews for both conventional and LP formulas are the same. The basic question is to determine how often formulas should be recomputed and changed. The optimum frequency of formulation is the frequency which minimizes the sum of the following two costs:

1. Computing and changeover costs.
2. Cost of using a formula not least-cost.

These costs are opposing costs. For example, computing and changeover costs increase as the frequency of review increases, but the cost of using a formula not least-cost decreases as the frequency increases.

The cost of using a formula not least-cost is the additional cost per ton of ingredients times the number of tons manufactured. The additional cost of ingredients is the cost per ton incurred by continuing to use an old formula instead of the current least-cost formula (table 2). Given that 1 week is the shortest feasible interval between reviews, no additional costs for ingredients would be incurred during the first week of use. During the second and successive weeks of use, an additional cost for ingredients would be incurred. For example, the additional cost of ingredients for the cattle supplement of firm A was $0.209

Table 1.--Gross savings per ton of feed resulting from the use of linear programming and weekly reviews of 6 feeds, firms A and B, 23 weeks, 1963

Feed and protein content	Number of formula changes	Gross savings attributable to --		
		Weekly review and use of LP	Weekly review	Use of LP
		Dollars		
Firm A:				
Cattle supplement (55 pct.)	0	.73	.24	.49
Layer supplement (40 pct.)	1	2.31	.30	2.01
Swine supplement (35 pct.)	1	2.65	.16	2.49
Average		1.89	.23	1.66
Firm B:				
Dairy feed (16 pct.)	1	1.40	.84	.56
Layer mash (16 pct.)	2	1.64	.16	1.48
Swine supplement (38 pct.)	0	1.45	.49	.96
Average		1.50	.50	1.00
Study average		1.70	.37	1.33

Table 2.--Additional cost per ton of ingredients incurred during any given week by use of an old formula instead of least-cost formula, 6 feeds, firms A and B, 1963

Feed	Week of use					
	1st	2nd	3rd	4th	5th	6th
	Dollars					
Firm A:						
Cattle supplement	0	.0209	.0475	.1010	.1244	.1246
Layer supplement	0	.0030	.0233	.0550	.0886	.1221
Swine supplement	0	.0285	.0466	.0605	.0901	.1058
Average	0	.0175	.0391	.0722	.1010	.1246
Firm B:						
Dairy feed	0	.0482	.1733	.2660	.3148	.4466
Layer mash	0	.0405	.0909	.1390	.1305	.1572
Swine supplement	0	.0182	.0493	.0848	.1468	.1650
Average	0	.0356	.1045	.1635	.1973	.2563
Study average	0	.0265	.0718	.1179	.1492	.1904

during the second week of use and $0.475 during the third. Note that the cost of ingredients rises at an increasing rate as the number of weeks of use increases.

Firm A was charged $10 per formula by a computer service. Based on this charge, the weekly costs of reviewing formulas for the six feeds at specified intervals are given in table 3. Review at 2-week intervals was indicated for three of the six feeds. Only one of the six feeds warranted weekly review, the common practice of many firms currently using LP. The need for periodic review of feeds with low sales volume was indicated by firm B, whose swine supplement produced in volumes of only 25 tons per week warranted review at 4-week intervals.

Break-even tonnages between reviews can be mathematically computed for any amount of formulation cost. 1/ General formulas and results for formulation costs of $10 and $15 are given in table 4. For example, the break-even point between reviews at 1- and 2-week intervals is 377 tons for a formulation cost of $10. Since the break-even point between reviews at 2- and 3-week intervals is 85 tons, a review every 2 weeks is indicated for feeds with weekly volumes between 85 and 377 tons. For a formulation cost of $15, the optimum range for the 2-week interval is between 128 and 566 tons. 2/

IMPACT OF LINEAR PROGRAMMING ON INGREDIENT USAGE RATES

Since LP indicates frequent formula changes, the practical difficulty of coordinating purchasing and production can arise. Severe shortages or overstocking of ingredients can result from abrupt changes in ingredient usage rates. For quick delivery, special purchases at premium prices may be required. The magnitude of these problems can be ascertained by comparing the composition of ingredients used in the least-cost formulas for the six feeds of firms A and B (tables 5 through 10).

1/ The break-even point between a review at intervals of k and k+1 weeks is computed by equating the following expressions and solving for T:

$$\frac{C + \sum_{i=1}^{k} P_i T}{k} = \frac{C + \sum_{i=1}^{k+1} P_i T}{k+1}$$

Where:

C = Cost per formula.

P_i = Cost of additional ingredients incurred during the ith week. (See table 2.)

T = Tons of feed manufactured per week.

k = Number of weeks in the interval between reviews.

This expression reduces to: $T = C/(kP_{k+1} - \sum_{i=1}^{k} P_i)$

2/ When part of the formulation cost is fixed regardless of the number of formulas reviewed, the problem becomes very complex. In general, nearly all formulas would be reviewed at the same time, for example, every 2 weeks for the study firms.

Table 3.--Weekly costs of reviewing formulas at intervals of from 1 to 6 weeks, 23-week period, firms A and B, 1963 1/

Feed and volume	Interval between reviews (weeks)					
	1	2	3	4	5	6
	Dollars					
Firm A:						
Cattle supplement (300 tons per week)	10.00	[8.13]	10.17	15.21	19.71	23.71
Layer supplement (300 tons per week)	10.00	[5.45]	5.96	8.60	12.19	16.27
Swine supplement (500 tons per week)	[10.00]	12.12	15.85	19.45	24.57	29.29
Total	30.00	[25.71]	31.99	43.27	56.39	69.21
Firm B:						
Dairy feed (150 tons per week)	10.00	[8.60]	14.43	20.80	26.08	32.90
Layer mash (50 tons per week)	10.00	6.01	[5.52]	5.88	6.01	6.32
Swine supplement (25 tons per week)	10.00	5.46	4.46	[4.41]	4.99	5.54
Total	30.00	[20.06]	27.79	31.08	37.08	44.76

1/ Based on a direct formulation cost of $10 and additional ingredient costs reported in table 2.

NOTE: Boxes indicate costs at optimum frequency of review.

Table 4.--Break-even tonnages per week by length of interval between reviews and formulation costs, firms A and B, 23 weeks, 1963 1/

Length of interval between reviews (weeks)	Formulation costs		
	C 2/	$10	$15
		Tons	Tons
1	∞ to C/.0265	∞ to 377	∞ to 566
2	C/.0265 to C/.1171	377 to 85	566 to 128
3	C/.1171 to C/.2554	85 to 39	128 to 58
4	C/.2554 to C/.3806	39 to 26	58 to 39
5	C/.3806 to C/.9161	26 to 11	39 to 16

1/ Based on the cost of additional ingredients reported in table 2.
2/ Cost per formula.

Table 5.--Usage rates for selected ingredients used in linear programming formulas for cattle supplement, firm A, 22 weeks, 1963

Ingredient	Formula					
	A	B	C	D	E	F
Number of weeks least-cost	1	4	3	4 & 7	2	1
	Percent					
Cottonseed meal	1.0	1.0	1.0	1.0	12.7	3.0
Distillers grain	0	0	4.6	0	0	0
Linseed meal ...	1.0	1.0	1.0	1.0	8.1	1.0
Molasses (hydrol)	4.0	2.5	.3	.3	.3	.3
44-percent soybean meal	0	0	0	18.7	0	0
50-percent soybean meal	52.6	52.3	49.9	35.8	47.5	42.9

Table 6.--Usage rates for selected ingredients used in linear programming formulas for layer supplement, firm A, 22 weeks, 1963

Ingredient	Formula					
	A	B	C	D	E	F
Number of weeks least-cost	3	3	1	1	5	4
	Percent					
Feather meal ...	6.0	6.0	6.0	4.3	4.0	4.2
Meat scrap	1.0	1.0	2.3	7.0	7.6	7.7
50-percent soybean meal	60.2	60.2	58.9	57.1	57.1	55.7
44-percent soybean meal	0	0	0	0	0	1.2

Table 7.--Usage rates for selected ingredients used in linear programming formulas for swine supplement, firm A, 22 weeks, 1963

Ingredient	Formula									
	A	B	C	D	E	F	G	H	I	J
Number of weeks least-cost	1	7	4	1	1	1	1	1	1	4
	Percent									
Alfalfa meal	5.0	5.0	5.0	5.0	5.0	5.0	5.0	9.1	5.0	9.3
Meat scrap	5.0	5.0	5.0	5.0	5.0	7.1	5.0	5.0	5.0	5.0
Molasses (hydrol)	4.0	3.0	2.3	1.3	1.9	2.8	2.8	.3	2.9	.3
44-percent soybean meal	49.9	51.3	56.9	53.0	53.2	50.4	55.9	53.5	56.6	55.0
50-percent soybean meal	6.5	5.0	0	0	0	0	0	0	0	0

Table 8.--Usage rates for selected ingredients used in linear programming formulas for dairy feed, firm B, 23 weeks, 1963

Ingredient	Formula								
	A	B	C	D	E	F	G	H	I
Number of weeks least-cost	1	1	1	6	1	5,1,&1	3	1&1	1&2
	Percent								
Cobs	7.1	5.6	6.1	4.8	1.5	1.0	1.3	.2	0
Corn	28.2	22.3	25.3	19.1	6.1	4.1	5.3	.8	0
D & D grains	.5	.5	7.7	14.4	2.4	1.3	2.0	.5	.5
Gluten feed	7.1	25.0	25.0	25.0	25.0	25.0	24.5	25.0	25.0
Hominy feed	10.2	18.2	0	0	0	1.6	0	6.5	7.6
50-percent soybean meal	12.2	9.7	4.0	0	0	0	0	0	0
Wheat shorts	18.3	7.5	16.6	19.9	43.0	45.0	45.0	45.0	45.0
Wheat bran	0	0	0	0	5.0	5.0	5.0	5.0	5.0

Ingredient usage rates for the formulas of firm B generally fluctuated more than those for the formulas of firm A. Since firm B's dairy feed and layer mash were complete feeds, their lower protein content allowed for a much wider range of substitution than the comparable supplement formulas of firm A (tables 8 and 9). For example, firm B's dairy feed required 9 formulas and its layer mash 16 formulas, in contrast to only 6 formulas each required for the cattle supplement and layer supplement of firm A. Specifications for the swine supplement of both firms were similar. Although the swine supplement of firm B required fewer formulas than that of firm A, the proportion of ingredients used in it fluctuated greatly (table 10).

At firm A, several instances of abrupt changes in ingredient usage rates would have made it difficult to implement the formulas. For example, the percentage of linseed meal used in the cattle supplement feed increased from 1 percent in formula D to 8.14 percent in formula E (for a 2-week period) and then returned to 1 percent in formula F (table 5). In this same feed, the percentage of cottonseed meal used increased from 1 percent in formula E to 12.71 percent in formula F (for 1 week only) and then dropped to 1 percent in formula D. The layer supplement formula exhibited greater stability, though there was some inverse movement between the usage rates for meat scrap and feather meal (table 6). Though the swine supplement formula changed frequently (10 times), changes in ingredient usage rates were not severe (table 10).

The combined effect of these changes on the total amounts of cottonseed meal, linseed meal, and meat scraps used by firm A are shown in figure 1. Note the difficulties in adjusting procurement and inventories that would have been encountered in week 10 and during weeks 15 through 17.

Several alternative measures can be taken to minimize the severity of fluctuations in ingredient usage rates arising from the implementation of LP formulas. One possibility is to delay implementation of the new formulas until inventory levels and procurement schedules can be adjusted to pending usage rates. Differences in the amount of delivery time required for different ingredients, however, limit the effectiveness of this approach. When delayed implementation is feasible, there may be a problem of providing the purchasing department with accurate projections of revised usage rates.

Another alternative is to include LP restrictions that limit fluctuations in ingredient usage rates. For example, the level of an ingredient in the LP specifications might be restricted to $\pm$ 5 percent of its current usage rate. This approach would permit gradual changes in ingredient usage rates over successive periods between reviews. These gradual changes may also be preferable from a feeding standpoint, since abrupt changes in taste, odor, or texture may "throw the animal off feed." Computational problems of changing specifications for each formulation, however, restrict the usefulness of this approach.

A third alternative is to anticipate changes in usage rates. Most LP programs have routines that can provide supplementary information on the sensitivity of the optimum solution. Through sensitivity analysis, changes in ingredient usage rates and their probable duration can be anticipated. Sensitivity analysis is discussed in the next section.

Table 9.--Usage rates for selected ingredients used in linear programming formulas for layer mash, firm B, 23 weeks, 1963

Ingredient	Formula															
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Number of weeks least-cost	1	1	1	2	1	1	1	1	1	1	2	1	1	2	4	2
	Percent															
Corn	45.8	59.4	55.8	46.4	50.1	59.5	63.9	46.4	46.2	47.0	45.8	47.0	51.2	45.8	45.8	45.3
Hominy	8.2	0	0	0	0	0	0	0	.4	0	0	0	0	0	0	0
Wheat shorts	5.0	5.0	5.0	15.0	15.0	5.0	5.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
Meat and bone	3.2	3.3	0	2.7	2.6	3.3	3.2	2.7	2.6	2.4	2.4	2.4	9.8	4.5	2.4	2.2
50-percent soybean meal	10.5	11.0	14.4	9.5	9.6	11.0	11.1	9.5	9.0	9.8	9.9	9.8	.5	6.2	9.4	8.5
Wafer meal	9.5	3.2	5.9	9.7	7.1	3.2	0	9.7	10.0	9.1	10.0	9.1	6.0	10.0	10.0	10.0

Table 10.--Usage rates for selected ingredients used in linear programming formulas for swine supplement, firm B, 1963

Ingredient	Formula					
	A	B	C	D	E	F
Number of weeks least-cost	2	7	1	1	11	1
	Percent					
Meat and bone	16.4	13.6	21.6	21.7	21.3	18.9
Feather meal	0	0	.6	0	0	5.0
Fish meal	0	4.0	0	0	1.0	0
44-percent soybean meal	24.3	0	39.7	45.2	54.6	49.1
50-percent soybean meal	34.5	57.5	14.9	10.0	0	0

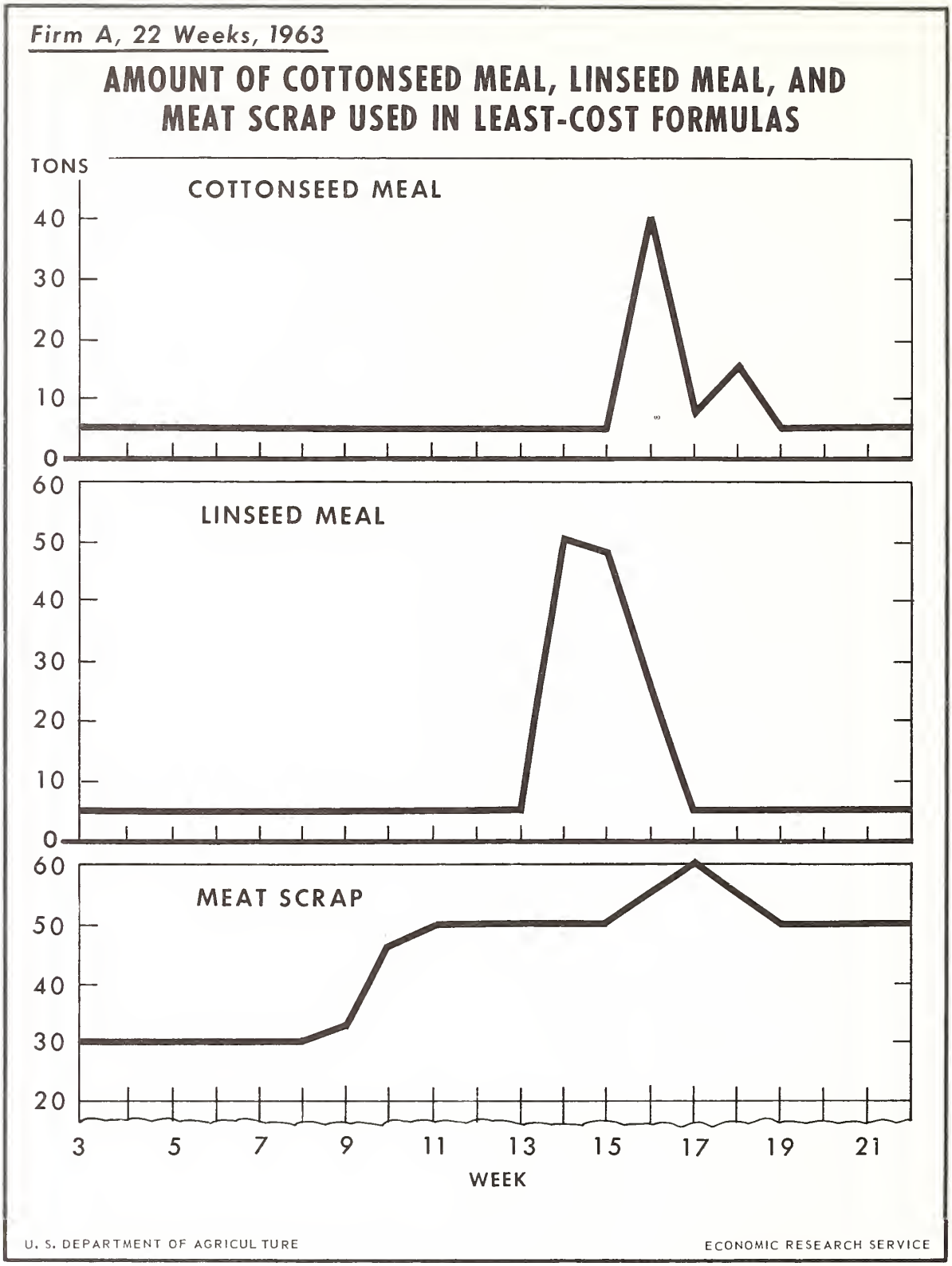
Firm A, 22 Weeks, 1963
AMOUNT OF COTTONSEED MEAL, LINSEED MEAL, AND MEAT SCRAP USED IN LEAST-COST FORMULAS
TONS
COTTONSEED MEAL
40
30
20
10
0
LINSEED MEAL
60
50
40
30
20
10
0
MEAT SCRAP
60
50
40
30
20
3
5
7
9
11
13
15
17
19
21
WEEK
U. S. DEPARTMENT OF AGRICULTURE
ECONOMIC RESEARCH SERVICE

Figure 1

SUPPLEMENTARY INFORMATION

Supplementary information from LP solutions can be useful to management if interpreted with care. 3/ Opportunity costs, the increase in cost resulting from the use of an ingredient not in the least-cost formula, and price ranges, the range over which ingredients in the formula remain least-cost, can help the purchasing department anticipate changes in ingredient usage rates. The cost of LP restrictions (shadow prices) can be used by the nutritionist to adjust formula specifications and pinpoint those areas needing top research priority.

Guidelines for Purchasing Ingredients

Price ranges and opportunity costs give the fluctuations in the price of each ingredient that could occur, provided all other prices remain constant, without changing the composition of the least-cost formula. For example, the price range for wheat middlings in table 11 ($36.04 to $42.76) indicates that it will be used at the same rate in the least-cost formula as long as its price is between $36.04 and $42.76, if all other prices remain stable. If the price stays well within this range and the rest of the market is stable, the buyer can be relatively certain that the price of wheat middlings will not cause the formula to be changed.

Special interpretation, however, is required for ingredients that are close substitutes, for example, 44-percent and 50-percent soybean meals (table 11). The price ranges of $74.11 to $75.60 for 44-percent meal and $80.15 to $81.93 for 50-percent meal are not helpful because both prices normally move together. The difference in the spread between their prices, rather than absolute prices, gives a more meaningful indication of probable change in usage rates. For example, a 50-cent increase in the spread between them would probably cause an increase in the use of 44-percent meal at the expense of 50-percent meal. Conversely, a decrease of 50 cents in the spread would probably cause an increase in the use of the 50-percent meal at the expense of 44-percent meal. By comparing the spreads between known substitutes with their price ranges, the buyer can anticipate variations in usage rates between reviews.

By watching price ranges and opportunity costs over time, one can determine general trends in the price sensitivity of ingredients. This information can be very helpful for making decisions on advance bookings. For example, the buyer can safely make a future commitment for an ingredient if the price ranges are wide. When price ranges are narrow, he can forego advanced booking in favor of spot purchasing.

To illustrate the interpretation of information from several periods, price ranges for selected ingredients are reported in table 12. For meat scraps, a change in ingredient usage rates in weeks 7 through 9 could have been predicted, since the price of the ingredient was below the lower range of the previous week in all three instances. Though the price of meat scraps

3/ For another discussion of this topic, see: Stafford, J.H., and Snyder, J.C. Interpreting Least-Cost Feed Formula Data. Purdue Agr. Expt. Sta., Res. Prog. Rpt. 93, Feb. 1964.

Table 11.--Price per ton of ingredients used in a least-cost cattle supplement formula, and the price range over which formula was not changed, firm A, 1963

Ingredients	Current price	Price range over which formula was not changed	
		Lower limit	Upper limit
	Dollars	Dollars	Dollars
Alfalfa meal	47.00	40.95	1/
Bran (wheat)	36.00	1/	39.46
Cottonseed meal	75.00	71.82	1/
Gluten feed	39.00	1/	48.77
Limestone	10.00	1/	21.26
Linseed meal	69.00	63.68	1/
Molasses (cane)	48.00	21.23	1/
Molasses (hydrol)	36.00	21.74	1/
Phosphorous (dical)	82.00	14.81	98.39
Salt	18.00	1/	21.37
44-percent soybean meal	74.50	74.11	75.60
50-percent soybean meal	81.50	80.15	81.93
Wheat middlings	39.50	36.04	42.76
Urea compound	100.00	1/	318.41

Ingredients not used	Opportunity cost per ton, or price reduction necessary for ingredient to be used
	Dollars
Beet pulp	31.56
Distillers grain	4.77
Fat	68.63
Phosphate	12.89

1/ Unbounded.

Table 12.--Current price and price ranges for layer supplement and cattle supplement formulas of firm A, 22 weeks, 1963

Week	Layer supplement				Cattle supplement							
	Meat scrap				Linseed				Cottonseed			
	Percentage used	Current price	Price range: Lower limit	Price range: Upper limit	Percentage used	Current price	Price range: Lower limit	Price range: Upper limit	Percentage used	Current price	Price range: Lower limit	Price range: Upper limit
	Percent	Dollars			Percent	Dollars			Percent	Dollars		
1	1.0	103.00	97.25	1/								
2	1.0	110.00	98.72	1/	1.0	78.00	68.34	1/	1.0	77.00	74.47	1/
3	1.0	110.00	101.67	1/	1.0	78.00	70.40	1/	1.0	77.00	76.88	1/
4	1.0	110.00	99.71	1/	1.0	76.00	69.03	1/	1.0	77.00	75.27	1/
5	1.0	105.00	99.71	1/	1.0	76.00	69.03	1/	1.0	77.00	75.27	1/
6	1.0	100.00	98.60	1/	1.0	73.00	67.84	1/	1.0	75.00	74.17	1/
7	2.4	98.00	96.17	98.60	1.0	73.00	67.84	1/	1.0	75.00	74.17	1/
8	7.0	95.00	93.31	95.69	1.0	73.00	67.84	1/	1.0	75.00	74.17	1/
9	7.6	90.00	82.15	93.29	1.0	71.00	66.12	1/	1.0	75.00	72.73	1/
10	7.6	88.00	81.96	93.27	1.0	69.50	65.12	1/	1.0	74.00	71.74	1/
11	7.6	88.00	81.59	93.22	1.0	68.50	63.13	1/	1.0	74.00	69.75	1/
12	7.6	87.00	81.59	93.22	1.0	64.50	63.13	1/	1.0	74.00	69.75	1/
13	7.6	86.00	81.59	93.22	8.1	60.50	59.88	61.82	1.0	74.00	68.15	1/
14	7.6	86.00	82.21	93.54	8.1	61.50	60.91	62.01	1.0	70.00	69.33	1/
15	7.6	86.00	82.06	93.28	1.0	64.00	61.96	1/	2.7	70.00	69.78	71.24
16	7.6	86.00	81.78	93.24	1.0	67.00	62.82	1/	1.0	70.00	69.95	1/
17	7.6	88.00	81.78	93.24	1.0	67.00	62.82	1/	1.0	70.00	69.95	1/
18	7.6	88.00	81.78	93.24	1.0	68.00	62.82	1/	1.0	74.00	69.95	1/
19	7.6	88.00	82.02	93.38	1.0	69.00	61.51	1/	1.0	74.00	69.15	1/
20	7.6	86.00	82.02	93.38	1.0	69.00	61.51	1/	1.0	72.00	69.15	1/
21	7.6	88.00	82.41	93.70	1.0	69.00	62.50	1/	1.0	75.00	70.14	1/
22	7.6	92.00	84.02	95.39	1.0	69.00	63.68	1/	1.0	75.00	71.82	1/

1/ Unbounded.

continued to decline until week 16, the ranges correctly indicated no increase in the usage rate. For linseed meal, the lower limit of $63.13 in week 12 was greater than the price of $60.50 in week 13. This correctly forecasted the increase in the usage rate that occurred in week 13. Similarly, the upper limit of $62.01 in week 14 indicated that the rise in price to $64 in week 15 would cause a reduction in its use.

On the other hand, the change in the usage rate for cottonseed meal during week 15 was more complicated. It was the combined effect of price changes for linseed meal and the soybean meals. 4/ Note, however, that the low upper limit of $71.21 in week 15 indicated that the high usage of linseed meal may be only temporary (table 12).

Guidelines for Formula Specifications

The cost of LP restrictions (shadow prices) reflects the change in the cost of the feed resulting from the alteration of formula specifications. A continuing analysis of the cost of LP restrictions is a valuable guide to those doing nutrition research. With a knowledge of the cost of different quality specifications, emphasis can be placed on narrowing the margins of safety on the more costly specifications while keeping wide margins on the less costly ones.

The cost per 1 percent change per ton in selected specifications for a swine supplement formula are given in table 13. Note the wide variation in the cost of different specifications. For example, the cost per 1 percent change per ton in the minimum specification for phosphorous was 343 cents, as compared with a cost of 3.41 cents per 1 percent change per ton in the minimum specification for meat scrap. Thus, the cost of the formula could be reduced 34.3 cents (.1 x 343) by reducing the minimum phosphorous specification by one-tenth of 1 percent. However, reducing the meat scrap specification by a full 1 percent would reduce the cost of the formula only 3.41 cents per ton.

A usage range on the specification cost is also available with most LP programs. The two columns at the extreme right of table 13 give the ingredient usage range over which the cost of the specification applies. For a minimum specification, a lower cost is incurred for reducing a specification below the lower limit, and a higher cost is incurred if the specification is raised above the upper limit. And, conversely, reducing a maximum specification below the lower limit increases the cost, but an increase above the upper limit lowers the cost.

The costs of specifications for a series of successive time periods, however, are required for meaningful interpretation. Tables 14, 15, and 16 show the average costs per 1 percent change per ton in the specifications for the three feeds of firm A, and price ranges during the study period. Some specification costs were relatively stable; others were very sensitive and unstable. In general, shifts in the costs of specifications followed patterns that reflected price movements of major ingredients in the feed.

4/ The price per ton of 50-percent meal was $78.50, $79.50, and $78 for weeks 15, 16, and 17, respectively. The price per ton of 40-percent meal was $72, $73.50, and $72 for weeks 15, 16, and 17, respectively.

Table 13.--Cost per 1 percent change per ton in selected specifications for a swine supplement formula, and the range of use within which these costs are applicable, firm A, 22 weeks, 1963

Specifications	Cost per 1 percent change in specification	Range of use over which costs are applicable	
		Lower limit	Upper limit
	Cents	Percent	Percent
Crude protein minimum	90.97	34.6	38.9
Calcium maximum	56.96	3.2	4.8
Phosphorous minimum	343.00	.9	2.2
Fish meal minimum	40.33	4.8	6.3
Fish solubles minimum	12.58	1.4	3.3
Meat scrap minimum	3.41	1.5	6.2
Molasses (cane) minimum	14.78	0	2.8
Salt maximum	13.64	1.2	2.0
Mineral mix	162.86	0	3.2
Wheat middlings maximum	9.73	4.9	11.8

Table 14.--Average cost per 1 percent change per ton in selected specifications for cattle supplement, and the price range, firm A, 22 weeks, 1963

Specifications	Average cost per 1 percent change in specification	Price range	
		Lower limit	Upper limit
	Cents	Cents	Cents
Crude protein minimum	105.94	80.47	113.38
Calcium maximum	51.62	26.88	73.58
Phosphorous minimum	347.51	329.53	367.39
Alfalfa meal minimum	9.08	4.75	16.74
Bran maximum	1.15	.16	3.46
Cottonseed minimum	2.37	0	5.85
Gluten feed maximum	8.00	2.06	11.09
Linseed meal minimum	5.29	0	11.25
Molasses (cane) minimum	16.28	6.97	26.77
Molasses (hydrol) maximum	.01	0	.15
Urea maximum	178.95	150.70	217.78
Bran and middlings maximum	6.86	0	10.00

Table 15.--Average cost per 1 percent change per ton in selected specifications for layer supplement, and the price range, firm A, 22 weeks, 1963

Specifications	Average cost per 1 percent change in specification	Price range	
		Lower limit	Upper limit
	Cents	Cents	Cents
Productive energy minimum (megacalorie)	.57	0	1.47
Crude protein minimum	100.13	89.10	107.12
Phosphorous minimum	252.88	121.29	389.19
Alfalfa meal minimum	14.17	3.38	24.19
Blood meal minimum	19.95	12.09	30.78
Fish meal minimum	39.45	33.03	45.04
Fish solubles minimum	20.25	14.49	25.27
Meat scrap minimum	1.92	0	11.27
Blood meal and feather meal maximum	3.57	0	18.24
Fat minimum	39.50	0	69.43
Salt maximum	2.84	0	8.78
Inorganic phosphorous minimum	.01	0	.02

Table 16.--Average cost per 1 percent change per ton in selected specifications for swine supplement, and the price range, firm A, 22 weeks, 1963

Specifications	Average cost per 1 percent change in specification	Price range	
		Lower limit	Upper limit
	Cents	Cents	Cents
Lysine minimum (.001)	.23	0	3.17
Methionine minimum (.001)	3.07	0	11.87
Crude protein minimum	77.68	61.75	90.97
Calcium maximum	68.62	55.87	73.73
Phosphorous minimum	333.62	329.41	343.88
Alfalfa minimum	5.21	0	9.31
Fish meal minimum	45.83	38.37	53.52
Fish solubles minimum	15.18	10.25	18.83
Linseed meal maximum	.16	0	3.55
Meat scrap minimum	8.18	0	21.98
Molasses (cane) minimum	8.68	5.42	15.16
Middlings maximum	10.70	0	17.99

APPENDIX

Formulas which Minimize the Cost of Feed Needed for Producing a Unit of Livestock

Linear programming can be used to find the formula which is least-cost per unit of weight, TDN, metabolizable energy, or per unit of any other single factor. Since feed is sold by weight, most firms prefer to find formulas that are least-cost per unit of weight. However, it is more desirable to find a formula that is least-cost per unit of livestock output, since units of TDN or metabolizable energy are generally better measures of livestock output than units of weight. Hence, a feed formula that is least-cost per unit of TDN or metabolizable energy will be more likely to minimize the cost of producing livestock than a formula which is least-cost per unit of weight.

For the study period, firm B's formula for dairy feed was also minimized per unit of TDN, and its layer mash was minimized per unit of metabolizable energy. The cost per ton of feed was about 3.0 percent less for the weight formulas than for the TDN or metabolizable energy formulas (table 17). Little difference existed in the cost per megacalorie of metabolizable energy of the two types of formulas for layer mash. For the dairy feed, the cost per 1 percent of TDN was 3.4 percent less for the TDN formulas than for the weight formulas. However, through the use of LP information on the cost (shadow price) of the TDN restriction of the weight formulas, the TDN minimum can be raised to the point that its cost is equal to the average cost per 1 percent of TDN. By this procedure, it is possible to have a formula that is for all practical purposes least-cost both in terms of TDN and weight.

The only major difference in the composition of the ingredients used in the two types of dairy formulas was that hominy feed used in the TDN formulas was substituted for corn and cob meal in the weight ones (table 8, p. 7 and table 18). Little difference existed in the composition of the ingredients used in the two types of layer mash formulas (table 9, p. 8 and table 19). Fewer inventory and procurement problems would have occurred with the metabolizable energy formulas, since they changed only 10 times, in contrast to the 16 times that the weight ones changed.

Table 17.--Cost of dairy feed and layer mash formulas based on weight, TDN, or metabolizable energy, firm B, 23 weeks, 1963

Item	Least-cost formulas based on--		Percentage change
	Weight	TDN, or metabolizable energy	
Dairy feed:			
Cost per ton (dollars)	49.35	50.88	3.1
Cost per 1 percent of TDN (cents)	68.68	66.38	-3.4
Percentage of TDN	71.85	76.64	0
Layer mash:			
Cost per ton (dollars)	54.18	55.96	3.3
Cost per megacalorie of energy (cents)	2.08	2.03	-.2
Calories per pound	1300	1383	0

Table 18.--Usage rates for selected ingredients used in dairy feed formulas that minimize cost per 1 percent of TDN, firm B, 23 weeks, 1963

Ingredient	Formula									
	A	B	C	D	E	F	G	H	I	J
Number of weeks least-cost	1&1	1	2	1&3	3	1	2,1, & 2	1	2	1&1
	Percent									
Cobs	0	0	0	0	0	0	0	0	0	0
Corn	0	0	0	0	0	0	0	0	0	0
D & D grains	8.8	2.7	14.2	14.4	14.3	14.2	0.5	10.3	15.2	7.8
Gluten feed	8.2	13.9	5.1	5.0	12.7	12.6	22.4	9.0	4.1	25.0
Hominy feed	50.0	50.0	28.8	28.3	38.3	38.5	21.4	25.0	25.0	40.3
50-percent soybean meal	5.3	9.5	.5	0	0	0	0	0	0	0
Wheat bran	0	0	0	0	0	0	0	0	0	0
Wheat shorts	20.0	7.5	41.2	41.7	24.0	24.0	45.0	45.0	45.0	16.2
TDN	78.0	77.6	77.1	77.1	77.1	77.1	75.4	76.6	77.0	76.4

Table 19.--Usage rates for selected ingredients used in layer mash formulas that minimize cost per calorie, firm B, 1963

Ingredient	Formula									
	A	B	C	D	E	F	G	H	I	J
Number of weeks least-cost	3,1, & 1	1	3	1&1	1&4	1	1,1, & 1	1	1	1
	Percent									
Corn	59.1	62.4	62.7	58.8	56.5	59.8	58.5	52.2	51.2	50.4
Hominy	0	0	0	0	0	0	0	0	0	0
Wheat shorts	5.0	5.0	5.0	5.0	8.1	5.0	6.7	15.0	15.0	15.0
Meat and bone	4.1	4.0	4.0	4.1	3.8	10.0	10.0	9.6	9.0	3.1
50-percent soybean meal	11.4	10.8	10.8	12.5	11.7	8.1	7.5	4.6	5.8	10.1
Wafer meal	10.0	7.0	6.7	10.0	10.0	10.0	10.0	10.0	10.0	10.0
Metabolizable energy (calories per lb.)	1388	1381	1380	1387	1379	1396	1392	1373	1366	1358

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